MINOR STUDIES FROM THE PSYCHOLOGICAL LABORATORY OF CORNELL UNIVERSITY

Communicated by E. B. TITCHENER and E. G. BORING

XXXVII. THE WEBER-FECHNER LAW AND SANFORD'S WEIGHT EXPERIMENT

By MYRL COWDRICK

In the Cornell Laboratory we have records ranging from 1906 to 1917 of 89 performances of Sanford's weight experiment[1] by 48 observers. Ten of these observers performed the experiment only once; 32 of them did it twice; four, three times; and one, four times. Thus of the 89 experiments[2], 43 were performed after an initial experiment, *i. e.,* after a certain amount of practice. The average weight (in grams) of every one of the five piles of envelopes was computed both for the total 89 experiments and for the 43 experiments performed after some practice. Since the five piles in this experiment mark off four equal sense-distances, we may arbitrarily call the intensities of sensation (S) for the successive piles, 0, 1, 2, 3, and 4. The corresponding averages of the stimulus (R, av. weight of the envelopes in a pile in grams) are then as follows:

S:	0	1	2	3	4
R (av. total 89 exps.):	7.80	16.97	31.71	53.53	84.11
R (av. 43 exps. after practice):	7.75	16.53	30.43	52.17	83.93

The ratios between these two sets of R's are respectively:

2.17, 1.92, 1.69, 1.57;
and 2.14, 1.84, 1.71, 1.61.

Since the ratios are in neither case constant the Weber-Fechner Law is not realized, although it is approximated. The continuous decrease of the ratio from the lighter to the heavier piles shows that the actual curve is one of less eccentricity than is the logarithmic curve of the Weber-Fechner formula.[3] One such less eccentric function is that

[1] E. B. Titchener, *Experimental Psychology,* vol. II, 1905, pt. i, 33f.; pt. ii, 82ff.; E. C. Sanford, *A Course in Experimental Psychology,* 1898, 340f., 413f.

[2] The cases preceding add up to 90, but the data for the first trial by one observer who did the experiment twice are missing.

[3] Of course we must not generalize for a range wider than the actual limits of our experiment. The decreasing ratios might increase with still heavier piles, as do Ebbinghaus' ratios for brightness: 2.3, 2.1, 2.1, 1.8, 1.7, 1.7, 2.0; H. Ebbinghaus, *Grundzüge der Psychologie,* I, 1905, 520.

proposed by Fullerton and Cattell,[4] in which S is proportional to the square root of R.[5] The general equations for these two functions are:[6]

Weber-Fechner	$S = k \log R + a$
Fullerton and Cattell:	$S = c\sqrt{R} + b$

We adjusted both sets of averages (the 89 cases and the 43 cases after practice) by the method of least squares[7] to these two equations, thus determining the most probable values of the constants $k, c, a,$ and b.

Since the ratios between the two lightest piles (2.17 and 2.14) depart furthest from constancy, it appears that the Weber-Fechner Law holds more closely for the four heavier piles. Therefore we repeated our computations, omitting the average for the lightest pile.

The most probable equations resulting from all eight adjustments by the method of least squares are given in Table I.

TABLE I

MOST PROBABLE EQUATIONS BY THE METHOD OF LEAST SQUARES

	Weber-Fechner $S = k \log R + a$	Fullerton and Cattell $S = c\sqrt{R} + b$
Average of total 89 experiments:		
5 intensities ($S = 0$ to 4)......	$S = 3.89 \log R - 3.64$	$S = .623\sqrt{R} - 1.616$
4 intensities ($S = 1$ to 4)......	$S = 4.40 \log R - 4.52$	$S = .592\sqrt{R} - 1.383$

[4] G. S. Fullerton and J. McK. Cattell, *On the Perception of Small Differences,* 1892, esp. 152ff.

[5] The formula of Fullerton and Cattell is simply a special case of the relation proposed by Plateau, Grotenfelt, and others, *viz.,* that equal R-ratios correspond to equal S-ratios. The formula for this hypothesis is $S = c\,R_p + b$. In the formula of Fullerton and Cattell, $p = 0.5$. See Titchener, *op. cit.,* pt. ii, 69; W. Wundt, *Physiologische Psychologie,* I, 1908, 640.

[6] The constants of proportionality are k and c. The constants a and b are so taken that the values of S come out in the arbitrarily assumed scale: 0, 1, 2, 3, 4. Obviously the value $S = 0$ is a fairly heavy weight, but the constants could readily be altered without readjusting the equations, for any other scale of S that might be desired. The constant a also includes the unknown value "$k \log r$," since the unit of R is taken here as the gram and not as the value of the stimulus limen, r; see Titchener, *op. cit.,* pt. ii, xxviii.

[7] We used a linear method of adjustment by least squares; see L. D. Weld, *Theory of Errors and Least Squares,* 1916, 73f., 89f., and, in general, 65-103.

An adjustment to Plateau's ratio-hypothesis (see note above) would necessarily have given a closer approximation, since there are three constants instead of two to be determined; but to be able to adjust with little error for three constants when there are only four or five observational values would not be positive evidence in favor of the hypothesis. Since the adjustment of an exponential equation is a laborious matter (*cf.* Weld, *op. cit.,* 178-180) and since the results would have little significance, we have not attempted it.

TABLE I—*Continued*

Average of 43 experiments after practice:		
5 intensities ($S = 0$ to 4)......	$S = 3.46 \log R - 3.01$	$S = .677 \sqrt{R} - 1.892$
4 intensities ($S = 1$ to 4)......	$S = 4.45 \log R - 4.56$	$S = .587 \sqrt{R} - 1.311$

We can measure the adequacy of the two hypotheses in all cases if we compute from every equation the theoretical values of R which that equation gives for every value of S, subtract this theoretical R from the R observed for the same S, square this difference, and then find the sum of these squares for all values of S. This sum of the squares of these deviations of observed values from theoretical values constitutes a measure of the degree with which an hypothesis is approximated. In the instances with which we are at present dealing the sums will, however, tend to be higher when the five squares for the five intensities are added than when the four squares for the four intensities are taken. Accordingly we have divided every sum by the number by which the normal equations are overdetermined (three or two)[7a], in order to get comparable values. These measures are given in Table II.

TABLE II

MEASURES OF THE DEGREE WITH WHICH THE TWO HYPOTHESES ARE APPROXIMATED BY THE OBSERVED DATA

Sums of the squares of the deviations of the average observed values from the theoretical values (computed from the least square equations of Table I), divided by the number of overdeterminations.

	Weber-Fechner	Fullerton and Cattell
Average of total 89 experiments:		
5 intensities.............	28.014	5.145
4 intensities.............	6.783	2.578
Average of 43 experiments after practice:		
5 intensities.............	20.180	25.527
4 intensities.............	3.562	5.223

From inspection of Table II we may draw for our data the following conclusions:

(1) In the complete experiment (all 89 cases, 5 intensities) the formula of Fullerton and Cattell represents the actual results much more adequately than does the Weber-Fechner Law (5.145 *vs.* 28.014).

(2) The limitation of the range of the experiment by the omission of the least intensity increases the approximation of the observed data

[7a] *Cf.* W. W. Johnson, *Theory of Errors and Method of Least Squares*, 1915, 108.

to both formulae, but still leaves a decided difference in favor of the formula of Fullerton and Cattell (2.578 *vs.* 6.783).

(3) With practice the observed results approach the form of the Weber-Fechner Law and depart from the function of Fullerton and Cattell, so that the former hypothesis now becomes the more representative (25.527 *vs.* 20.180).[8]

(4) With the more limited range of intensities after practice the approximation to both hypotheses is greatly improved, as it was without practice; and the Weber-Fechner Law remains the more representative hypothesis, as it was after practice for the full range.

(5) If the equations of Table I are plotted together with the curves of the observed values, it will be found that the curve of actual values lies always between the two hypotheses; with practice, nearer the logarithmic curve; without practice, nearer the curve of the square root of the magnitude. Thus it appears that the formula of Fullerton and Cattell deviates from the logarithmic function of the Weber-Fechner Law in the proper direction, but deviates too far. Whether after great practice with Sanford's weights any correction of the Weber-Fechner formula in the direction of the equation of Fullerton and Cattell would be necessary, cannot be said.[9]

XXXVIII. An Example of the Fractionation of Data From the Method of Constant Stimuli for the Two-Point Limen

By L. B. Hoisington

We present herewith the results of a determination of the limen of dual impression upon the skin, by the method of constant stimuli,[1] in which the data have been fractionated and dealt with separately for every fraction. Our problem is analogous to that undertaken by Fernberger for lifted weights (*The Effects of Practice in Its Initial Stages in Lifted Weight Experiments and Its Bearing upon Anthropometric Measurements*).[2] We have, however, fractionated more completely than Fernberger, and have thus been able to extend our treatment to more general conclusions.[3] On the other hand, we have intended our case to be merely an illustration of the manner in which such data may be treated, for we used but two observers; whereas Fernberger had ten subjects.

The experimental work was performed by A. M. Palmer and P. R. Dickinson, both of whom accepted opportunities to enter the National Service at a time when the computation of their results was

[8]Titchener notes that the Weber-Fechner law is approached with practice; *op. cit.*, pt. ii, 83.

[9]We have noted that Plateau's law will approximate the observations, but that with so few determinations this approximation indicates little. Merkel's law (*cf.* Titchener, *op. cit.*, pt. ii, 69; Wundt, *op. cit.* I, 635 ff.), the straight line, is obviously inappropriate.

[1] E. B. Titchener, *Experimental Psychology*, II, 1905, i, 92-104; ii, 248-258; E. G. Boring, this Journal, 28, 1917, 280-293.

[2] S. W. Fernberger, this Journal, 27, 1916, 261-272.

[3] See Boring, this Journal, 27, 1916, 315-319; Fernberger, *Psychol. Bull.*, 14, 1917, 110-113; Boring, this Journal, 28, 1917, 280-293.